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Effective parameters representing a density-dependent pion–nucleus potential

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Abstract

We have shown that the s-wave pion–nucleus potential with medium-modified density-dependent parameters, such as $b_0(\rho)$ and $b_1(\rho)$, can be represented by a conventional form with constant parameters by replacing the nuclear density ($\rho(r)$) with an effective density, $\rho_e \approx 0.60\,\rho(0)$. The parameters in the conventional Ericson–Ericson potential are thus interpreted as being effective ones containing density-dependent effects.

The so-called "anomalous s-wave repulsion" in the pion–nucleus interaction has been a long-standing problem in pion physics. Many authors [1] have claimed the existence of a large extra repulsion, arising from the term that is quadratic in density, Re $B_0\rho(r)^2$, in the Ericson–Ericson potential [2,3]:

$$V(r) = U_{\rm s}(r) + U_{\rm p}(r), \tag{1}$$

$$U_{\rm s}(r) = -(2\pi/m_\pi) \times \left[\epsilon_1\big(b_0\rho(r) + b_1\Delta\rho(r)\big) + \epsilon_2 B_0\rho(r)^2\right],$$

$$\rho(r) = \rho_p(r) + \rho_n(r), \tag{2}$$

$$\Delta\rho(r) = \rho_n(r) - \rho_p(r), \tag{3}$$

where $U_{\rm s}(r)$ and $U_{\rm p}(r)$ are for the s-wave and p-wave parts, respectively, and $\rho_p(r)$ and $\rho_n(r)$ are the proton and neutron density distributions, respectively; also, $\epsilon_1 = 1 + m_\pi/M_N$ and $\epsilon_2 = 1 + m_\pi/2M_N$ (hereafter, we adopt units of m_π^{-1}, m_π^{-1} and m_π^{-4} for b_0, b_1 and B_0, respectively). Recently, the possibility of an anomalous repulsion in the isovector part (b_1) as a unique signature of chiral symmetry restoration was pointed out [4,5]. In this respect, the deeply bound π^- states [6–8] which were produced in heavy nuclei (in ^{207}Pb [9–12], ^{205}Pb [13,14] and Sn isotopes [15]) are of particular importance, since the binding energies and widths of the 1s π^- states depend nearly entirely on the s-wave potential ($U_{\rm s}$) and, thus, these 1s states in heavy nuclei ($N > Z$) provide key information on the isovector part of the s-wave potential. In fact, the binding energies and widths of the 1s π^- states in ^{205}Pb and ^{115,119,123}Sn indicated an enhanced value of the isovector parameter b_1 (over its free value

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of $-(90.0 \pm 1.6) \times 10^{-3}$ [16,17]): $b_1/b_1^{\text{free}} = 1.28$, which is ascribed to a decreased order parameter of the chiral symmetry breaking in the nuclear medium: $f_\pi^{*2}/{f_\pi}^2 = b_1^{\text{free}}/b_1 = 0.78 \pm 0.04$ [14,15]. Recent global fits of pionic atom data by Friedman [18,19] are consistent with this result.

Since the potential parameters are nuclear-density dependent in the theoretical context of Weise [4], a question can be raised as to the possible equivalence of the conventional Ericson–Ericson potential with constant parameters (model C) and a medium-modified density-dependent (model DD) potential. In the present note we clarify that the two models are equally valid and are connected to each other, since a density-dependent term can be effectively linearized.

First, we examine which part (r) of the nuclear density ($\rho(r)$) is probed by a bound π^-. Two typical nuclei, ^{16}O and ^{208}Pb, are considered. The former possesses only shallow bound states, whereas the latter accommodates halo-like deeply bound states. We take known potential parameters for the p-wave parts [1] and a set of s-wave parameters ($b_0 = -0.028$, $b_1 = -0.12$, $\text{Re}\, B_0 = 0$ and $\text{Im}\, B_0 = 0.055$). The proton and neutron density distributions are assumed to take 2-parameter Fermi distributions with known parameter values [14]. Fig. 1 shows the π^- densities

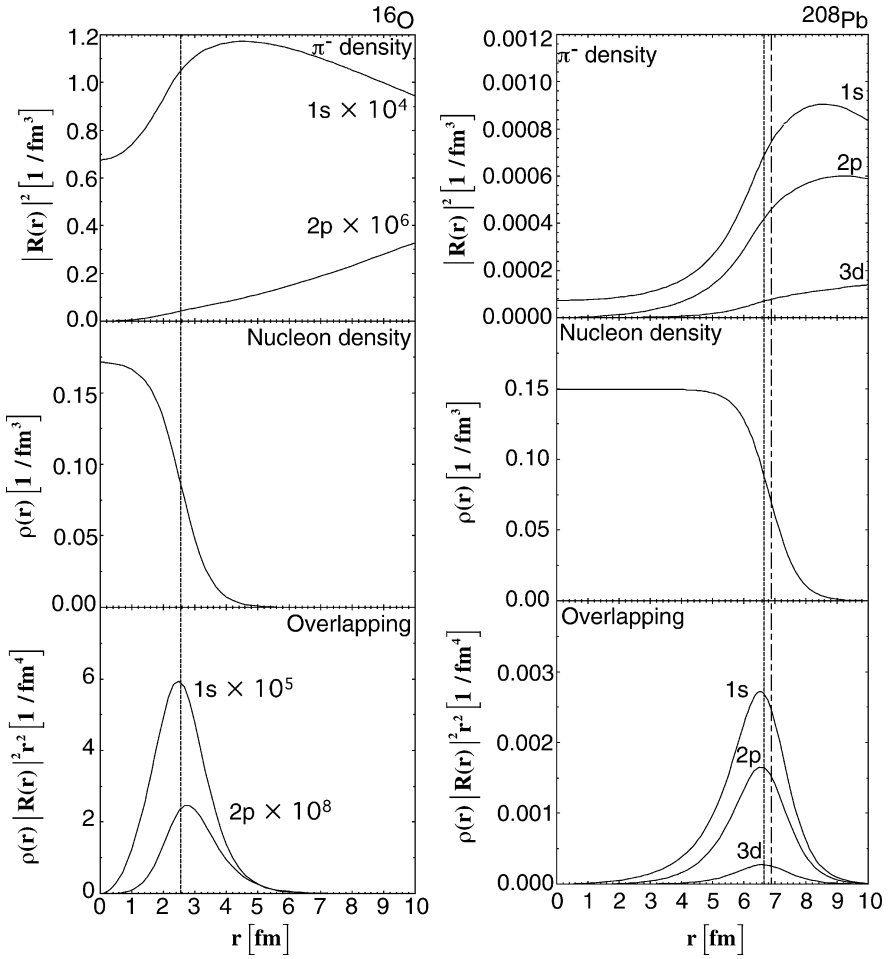

Fig. 1. Overlapping probabilities (lower frame) of the π^- densities (upper frame) with the nucleon densities (middle frame) in typical pionic bound states: (left) ^{16}O; (right) ^{208}Pb. The vertical broken lines show the half-density proton radii and the vertical dash-dotted line is for the half-density neutron radius in ^{208}Pb.

($|R_{\rm nl}(r)|^2$), the nuclear densities ($\rho(r)$) and the overlapping densities (namely, the nuclear densities probed by π^-), defined as

$$S(r) = \rho(r)|R_{\rm nl}(r)|^2 r^2, \tag{4}$$

for ^{16}O (1s and 2p) and ^{208}Pb (1s, 2p and 3d). From these figures we notice that the overlapping density is peaked at a radius slightly less than the half-density radius, *nearly independent of* the nucleus and the π^- quantum numbers. This means that the bound π^- effectively probes a fraction of the full nuclear density ($\rho_0 = \rho(0)$),

$$\rho_e \approx 0.60\,\rho_0. \tag{5}$$

This is a key to intuitively understanding the following results of numerical calculations. It should also be mentioned that, though any bound pion probes the nuclear density at $\rho \approx 0.6\,\rho_0$, the p-wave part of the potential is nearly inactive in the 1s π^- states, whereas both the repulsive s-wave and attractive p-wave potentials contribute in opposite directions to the binding energies of the 2p, 3d and higher-l states.

Seki and Masutani [20] emphasized the presence of a strong correlation between b_0 and $\mathrm{Re}\,B_0$ in describing the pionic-atom binding energies and widths, which we call the *Seki–Masutani relation*. Toki et al. found that the same correlation also exists for the deeply bound 1s and 2p states in ^{208}Pb [7,8]. Recently, this correlation has been revisited, and the following common relation has been established for the 1s states both theoretically and empirically [14]:

$$b_0^* \equiv b_0 + \frac{\epsilon_2}{\epsilon_1}\rho_e\,\mathrm{Re}\,B_0 \approx b_0 + 0.22\,\mathrm{Re}\,B_0 = \text{constant}. \tag{6}$$

This means that the binding energies (and widths) are nearly unchanged by varying either b_0 or $\mathrm{Re}\,B_0$, as long as these parameters are moved together so as to fulfill the above SM relation. In other words, neither b_0 nor $\mathrm{Re}\,B_0$ can be determined uniquely from the binding energies; it is only their combination, b_0^*, that can be precisely determined. The coefficient on $\mathrm{Re}\,B_0$ in (6) is slightly smaller for the higher states, and thus a global fit of shallow pionic atoms may yield a value of $\mathrm{Re}\,B_0$.

Various global fits gave a considerable spread of values for b_0 and $\mathrm{Re}\,B_0$ with large uncertainties. For instance, the best-fit values of $\{b_0,\ \mathrm{Re}\,B_0\}$ are

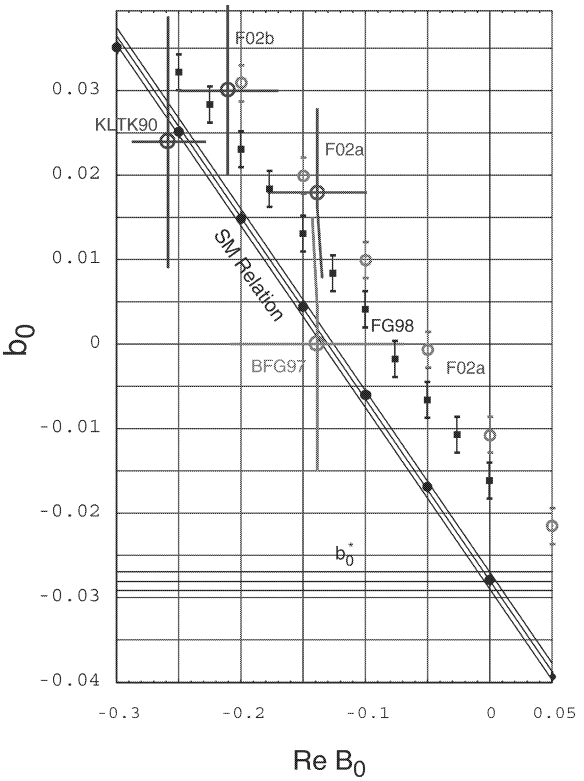

Fig. 2. Seki–Masutani relation between b_0 and $\mathrm{Re}\,B_0$. Best-fit values of b_0 versus $\mathrm{Re}\,B_0$ as a gridding variable, obtained in χ^2 minimization using the 1s pionic atom data in 6 $N = Z$ nuclei, are shown by closed circles, whose sizes are equal to the fitting uncertainties. They lie on the SM lines: $b_0^* = b_0 + 0.215\,\mathrm{Re}\,B_0 = -0.0280 \pm 0.0010$. The $\{b_0, \mathrm{Re}\,B_0\}$ parameters obtained from global fits of KLTK90 [21], BFG97 [1], F02a [18] and one of F02b [19] are shown by the large open circles with both vertical and horizontal error bars. Also shown are $\{b_0, \mathrm{Re}\,B_0\}$ sets from FG98 [22] (closed squares) and F02a [18] (small open circles), without conversion from b_0 to $b_0^{\rm F}$ in their convention.

$\{0.024(15), -0.26(3)\}$ in Konijn et al. (KLTK90) [21] and $\{0.000(15), -0.14(7)\}$ in Batty et al. (BFG97) [1]. With a different definition of b_0, namely, $b_0^{\rm F} = b_0 - \Delta b_0^{\rm DS}$, in which the double-scattering term ($\Delta b_0^{\rm DS}$, to be discussed later) is subtracted, Friedman obtained three different sets for $\{b_0^F,\ \mathrm{Re}\,B_0\}$: $\{0.018(10),\ -0.14(4)\}$ (F02a) [18], $\{0.030(10),\ -0.21(4)\}$ (F02b) [19] and $\{0.020(10),\ -0.15(4)\}$ [19]. These values, shown by the large open circles with both vertical and horizontal error bars in Fig. 2, are split beyond the quoted errors. Such a consequence of the global fits must be an artifact from regarding b_0

and Re B_0 as being independent uncorrelated parameters. It is to be noted that their b_0's (after subtraction of the double scattering term) take large positive values, indicating a strong isoscalar attraction, in contrast to the nearly vanishing $b_0^{\rm free}$ ($(1.7 \pm 1.0) \times 10^{-3}$ [16,17]).

On the other hand, the value of b_0 with a gridding Re B_0 can be precisely determined by χ^2 fits of the 1s π^- binding energies in six light symmetric nuclei (^{12}C, ^{14}N, ^{16}O, ^{20}Ne, ^{24}Mg and ^{28}Si), in which the isovector part ($b_1 \Delta\rho$) as well as the p-wave potential are inactive and, thus, the isoscalar parameters can be unambiguously determined. The values obtained in this fashion are in excellent agreement with the Seki–Masutani relation; they are shown by closed circles in Fig. 2 (their sizes correspond to the error bars), yielding a precise value of

$$b_0^* \equiv b_0 + 0.215\,\mathrm{Re}\,B_0 = -0.028 \pm 0.001. \tag{7}$$

Similar plots with a gridding Re B_0 using all of the pionic atom data in global fits were made by Friedman and Gal (FG98) [22] (shown by closed squares) and Friedman (F02a) [18] (small open circles). Their error bars are large, presumably because of extra uncertainties from their large parameter spaces including the p-wave parameters. The data points of the two analyses are different from the present one, partially because of the different definitions of b_0, as mentioned above. Nevertheless, the two data sets clearly indicate linear relations parallel to the SM correlation. It is to be noted that, if we compose b_0^* from the widely distributed values of b_0 and Re B_0 in each of KLTK90, BFG97, F02a and F02b, the b_0^* value is close to $-0.030 \sim -0.034$ (after subtracting the double scattering term in the case of F02a and F02b), which is not as much distributed as the individual values of b_0 and Re B_0. These values move even closer to -0.028 when we notice that these analyses took into account the so-called angle transformation term, as discussed later.

Seki and Masutani [20] showed that the correlation between b_0 and Re B_0 can be understood by replacing the term quadratic in density as $\langle \rho(r)^2 \rangle \to \rho_e \langle \rho(r) \rangle$, which is expected to hold for any pionic atom state with a common value of the effective density, $\rho_e \approx (1/2)\rho_0$ (*Seki–Masutani ansatz*). We understand that the localization of the overlapping densities, $S(r)$, near the half-density radii, as shown in Fig. 1, is the key to justify this ansatz.

We now verify the applicability of the SM ansatz to various functional forms of the potential by numerical calculations. Let us take a term, $F[\rho(r)]\rho(r)$, in the real part of the s-wave pion–nucleus potential, where $F[\rho(r)]$ is a density-dependent (DD) functional coefficient. We solved the Klein–Gordon equation with a DD potential and compare the numerical results with those without invoking DD, thereby examining how the DD potential is transposed as

$$F[\rho(r)]\rho(r) \to \overline{F}(\rho_e)\rho(r), \tag{8}$$

where $\overline{F}(\rho_e)$ is an effective constant parameter involving an effective nuclear density, ρ_e. Namely,

$$\overline{F}(\rho_e) = \frac{\langle F[\rho(r)]\rho(r) \rangle}{\langle \rho(r) \rangle}. \tag{9}$$

In the following we examine two important cases.

(i) *Density-dependent isovector term.* We take the following form according to Weise [4]:

$$\text{DD:} \quad b_1(r)\Delta\rho(r) = \frac{b_1^{\rm free}}{1 - \alpha\rho(r)}\Delta\rho(r). \tag{10}$$

The corresponding term in a conventional potential with a constant parameter involving an effective density (ρ_e) is

$$\text{C:} \quad \bar{b}_1\Delta\rho(r) = \frac{b_1^{\rm free}}{1 - \alpha\rho_e}\Delta\rho(r). \tag{11}$$

Numerical calculations of the 1s binding energy of ^{208}Pb with the DD term yielded a relation between B_{1s} and α, as shown in Fig. 3. On the other hand, the standard procedure assuming b_1 to be a constant ($\bar{b}_1$) gave another relation between B_{1s} and $1 - b_1^{\rm free}/\bar{b}_1$. The two relations are found to be nearly identical, if we take $\bar{b}_1$ with $\rho_e \approx 0.090\ \mathrm{fm}^{-3} \approx 0.60\,\rho_0$.

(ii) *Double-scattering isoscalar term.* Since $b_0^{\rm free}$ is nearly zero, double scattering gives a major cor-

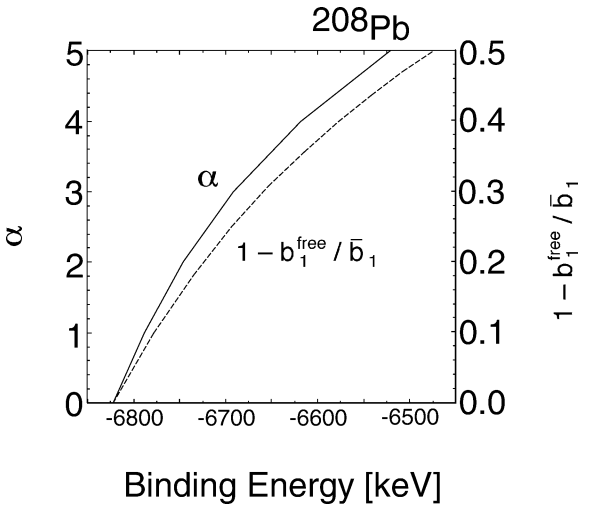

Fig. 3. The 1s binding energy calculated with the DD potential, (10), as a function of α, compared with that in a conventional potential with $\bar{b}_1$ as a constant parameter.

rection to the isoscalar term, as given by [2,3,23]

$$\text{DD:}\quad \Delta b_0^{\rm DS}\rho(r) = -\left[\left(b_0^{\rm free}\right)^2 + 2\left(b_1(\rho)\right)^2\right] \times \frac{3}{2\pi}k_F(\rho)\rho(r), \tag{12}$$

where $k_F(\rho) = [(3\pi^2/2)\rho(r)]^{1/3}$. We evaluated the full DD double-scattering effect with a density-dependent $b_1(\rho)$. The same form as (10) with α as a running parameter was adopted. The results were compared with those calculated with $\bar{b}_1$ as a constant:

$$\text{C:}\quad \Delta\bar{b}_0^{\rm DS}\rho(r) \approx -1.52\,\bar{b}_1^2\rho(r). \tag{13}$$

An equivalency between DD and C was found for both ^{16}O and ^{208}Pb, when $\rho_e \approx 0.60\,\rho_0$.

An equivalence between the DD potential and the conventional potential is also seen in Fig. 3 of [19], which shows nearly perfect agreements between the model-DD and model-C results concerning the calculated binding energies and the widths of the individual pionic states.

The present procedure can be applied to any kind of additional potential term. For instance, the effect of the so-called RIA correction, which was examined in global fits by Friedman [18,19], was found to be equivalent to invoking an additional isoscalar term, $\Delta b_0^{\rm RIA} \approx -0.021$. This can be ascribed to a reduction of Re B_0 by ≈ -0.1.

Finally, we make some comments on the so-called angle transformation (AT) term [1], which was taken into account in the global fits of BFG97, FG98, F02a and F02b, but not in the present Letter nor in [14]. This correction behaves like an s-wave potential, though it originates from the p-wave parameters. We found that including this correction is equivalent to making the following changes:

$$\Delta b_0^{\rm AT} \approx +0.003, \tag{14}$$

for both ^{16}O and ^{208}Pb, and

$$\Delta\,\mathrm{Im}\,B_0^{\rm AT} \approx \begin{cases} -0.008 & \text{for } ^{16}\mathrm{O}, \\ -0.014 & \text{for } ^{208}\mathrm{Pb}. \end{cases}$$

In other words, this correction gives rise to a small increase in the attraction of the real term and a small decrease in the imaginary term. Thus, the best-fit parameters (b_0^*) with and without this correction have differences, as given by (14). A small, but significant, difference in b_0^* between the global fits of BFG97, F02a and F02b ($-0.030 \sim -0.034$) and the present analysis ($-0.028(1)$) may be attributed to this correction. A similar change is caused on the isovector part: $\Delta b_1^{\rm AT} \approx +0.003$, but this is not significant compared with the large value of $|b_1|$.

In summary, we have shown that the Seki–Masutani ansatz can be generalized for any functional form of a density-dependent potential, namely, a DD term, $F[\rho(r)]\rho(r)$, can be represented by a corresponding density-linear term, $F[\rho_e]\rho(r)$, where ρ_e is an effective parameter. Therefore, the constant parameters in the conventional potential (C) are regarded as being density-dependent parameters at $\rho \approx \rho_e \approx 0.60\,\rho_0$. The reason for this relation can be readily understood from the fact that the overlapping density of any bound π^- with the nuclear density is peaked at a radius which is slightly smaller than the half-density radius. The above consideration means that the observed enhancement of b_1 (interpreted as a reduction of the chiral order parameter) in ^{205}Pb [14] and Sn isotopes [15], $R = b_1^{\rm free}/b_1 = f_\pi^*(\rho_e)^2/{f_\pi}^2 = 0.78 \pm 0.04$, is for the effective nuclear density of $\rho_e \approx 0.60\,\rho_0$, and thus a reduction, $f_\pi^*(\rho_0)^2/{f_\pi}^2 = 0.63 \pm 0.06$, would occur for the full nuclear density.

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